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双分数跳-扩散过程下再装期权定价模型

吴江增[1],王秋莹[2],薛　红[1]

(1.西安工程大学 理学院,陕西 西安 710048;2.华侨大学,福建 泉州 362000)

摘要:假设标的资产服从双分数布朗运动和跳过程驱动的随机微分方程,借助双分数布朗运动和跳过程随机分析理论,建立比分数布朗运动更一般的双分数跳-扩散过程下金融市场数学模型.运用保险精算方法研究再装期权定价问题,获得更一般的双分数跳-扩散过程下再装期权定价公式.

关键词:双分数布朗运动;跳-扩散过程;再装期权;保险精算

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Reload option pricing model under bifractional jump-diffusion process

WU Jiangzeng[1],*WANG Qiuying*[2],*XUE Hong*[1]

(1.School of Science, Xi′an Polytechnic University, Xi′an 710048,China;
2.Huaqiao University,Quanzhou 362000,Fujian,China)

Abstract:Assume that stock price follows the stochastic differential equation driven by the bifractional Brownian motion and jump process, the financial mathematical model under bifractional jump-diffusion process is built by the stochastic analysis theory of the bifractional Brownian motion and jump process. The reload option is discussed by using the actuarial approach, and the reload option pricing formula is obtained.

Key words:bifractional Brownian motion;jump-diffusion process; reload option;actuarial mathematics

0　引　言

期权定价是当今金融市场的热点问题,随着金融市场的飞速发展,标准期权已经不能满足金融市场的需要,因此各种新型期权逐渐进入复杂的金融市场, 而再装期权就是其中一种. Johnson 等[1]指出再装期

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通讯作者:薛红(1964—),男,山西省万荣县人,西安工程大学教授,研究方向为随机分析与金融数学.
E-mail:xuehonghong@sohu.com
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权是一种新型的欧式看涨期权，并且研究了几何布朗运动下的再装期权定价问题，给出了几何布朗运动条件下再装期权定价公式.文献[2]首次提出保险精算的方法，利用此方法解决了各种欧式期权定价问题并给出其定价公式. 关于保险精算方法的实际应用可见文献[3-8].文献[9-10]建立分数布朗运动环境下金融市场模型，利用保险精算的方法给出了分数布朗运动条件下的再装期权定价公式. 文献[11]运用鞅方法推导出了股票价格服从跳-扩散过程下再装期权的定价公式. 文献[12-16]讨论了双分数布朗运动概念、性质及应用，并指出双分数布朗运动是比分数布朗运动更一般的高斯过程.由于双分数布朗运动既不是半鞅也不是马尔可夫过程，不具备鞅性与马尔可夫性且不具有平稳独立增量性，使得双分数布朗运动比分数布朗运动更具有广泛性. 本文利用双分数布朗运动和跳过程建立金融市场数学模型，利用保险精算的方法和随机分析理论，研究再装期权定价问题并给出再装期权定价公式.

1　双分数跳-扩散过程下金融市场数学模型

定义 1[12]　称满足均值函数 $\mathrm{E}(B_t^{H,K})=B_0^{H,K}=0$ 的高斯过程 $\{B_t^{H,K},t\geqslant 0\}$ 为双分数布朗运动，如果协方差函数为

$$\mathrm{E}[B_t^{H,K}\,B_s^{H,K}]=\frac{1}{2^K}[(t^{2H}+s^{2H})^K-|\,t-s\,|^{2HK}],\quad t,\,s\geqslant 0,$$

其中 $H\in(0,1)$，$K\in(0,2)$.

当 $K=1$ 时，双分数布朗运动即为参数为 $H\in(0,1)$ 的分数布朗运动；特别地，当 $K=1$，$H=\dfrac{1}{2}$ 时，双分数布朗运动即为标准布朗运动.关于双分数布朗运动相关性质和随机分析基本理论见文献[12-15].

假设金融市场有两种资产，即无风险资产与风险资产，其价格方程分别满足随机微分方程

$$\mathrm{d}M_t=rM_t\mathrm{d}t,\tag{1}$$

$$\mathrm{d}S_t=S_t[\mu\mathrm{d}t+\sigma\mathrm{d}B_t^{H,K}+(\exp(J(t))-1)\mathrm{d}Q_t].\tag{2}$$

其中 M_t 表示无风险资产价格，S_t 表示风险资产价格，μ 为风险资产价格的期望收益率，σ 为无风险资产的波动率，$\{B_t^{H,K},t\geqslant 0\}$ 为定义在完备概率空间$(\Omega,\mathscr{F},P)$上的双分数布朗运动，Q_t 表示标的资产价格在时间区间$[0,t]$内的跳跃次数，且$\{Q_t,t\geqslant 0\}$服从参数为 λ 的 Poisson 过程，$\{J(t_i),i\geqslant 1\}$为独立同分布序列，且 $J(t_i)\sim N\left(-\dfrac{\sigma_J^2}{2},\sigma_J^2\right)$，$\exp(J(t_i))-1$ 表示标的资产价格在时刻 t_i 的相对跳跃幅度，并且假定 $\{B_t^{H,K},t\geqslant 0\}$，$\{J(t_i),i\geqslant 1\}$ 相互独立.

引理 1　随机微分方程(2)的解为

$$S_t=S_0\exp\left\{\mu t-\frac{1}{2}\sigma^2t^{2HK}+\sigma B_t^{H,K}+\sum_{i=1}^{Q_t}J(t_i)\right\}.\tag{3}$$

证明　当在$[0,t]$内没有发生跳跃时，由双分数 Ito 公式可得

$$S_t=S_0\exp\left\{\mu t-\frac{1}{2}\sigma^2t^{2HK}+\sigma B_t^{H,K}\right\}.$$

假定在 $t_1\in[0,t]$ 时刻内只发生一次跳跃，则在$[0,t_1)$内有

$$S_{t_1^-}=S_0\exp\left\{\mu t_1-\frac{1}{2}\sigma^2t_1^{2HK}+\sigma B_{t_1}^{H,K}\right\},$$

同理在$(t_1,t]$内有

$$S_t=S_{t_1}\exp\left\{\mu(t-t_1)-\frac{1}{2}\sigma^2(t^{2HK}-t_1^{2HK})+\sigma(B_t^{H,K}-B_{t_1}^{H,K})\right\}.$$

由式(2)有

$$S_{t_1}-S_{t_1-\frac{1}{n}}=\mu\int_{t_1-\frac{1}{n}}^{t_1}S_{u-}\mathrm{d}u+\sigma\int_{t_1-\frac{1}{n}}^{t_1}S_{u-}\mathrm{d}B_u^{H,K}u+\int_{t_1-\frac{1}{n}}^{t_1}(\exp(J(u))-1)S_{u-}\mathrm{d}Q_u,$$

当 $n\to+\infty$ 时，可得 $S_t-S_{t_1}=S_{t_1}(\exp(J(t_1))-1)$，所以

$$S_t=S_0\exp\left\{\mu t-\frac{1}{2}\sigma^2t^{2HK}+\sigma B_t^{H,K}+J(t_1)\right\}.$$

当跳跃次数服从 Poisson 过程时,可得

$$S_t = S_0 \exp\left\{\mu t - \frac{1}{2}\sigma^2 t^{2HK} + \sigma B_t^{H,K} + \sum_{i=1}^{Q_t} J(t_i)\right\}.$$

定义 2[9] 价格过程$\{S_t, t \geqslant 0\}$在$[0,t]$时间段内的期望收益率β_u, $u \in [0,t]$定义为

$$\exp\left\{\int_0^t \beta_u \mathrm{d}u\right\} = \frac{\mathrm{E}[S_t]}{S_0}. \tag{4}$$

引理 2 $\{S_t, t \geqslant 0\}$在$[t,T]$上的期望收益率为$\beta_u = \mu, u \in [0,t]$.

证明 由引理 1 知,在$[t,T]$内跳跃次数为 0 时,则

$$S_t = S_0 \exp\left\{\mu t - \frac{1}{2}\sigma^2 t^{2HK} + \sigma B_t^{H,K}\right\},$$

所以

$$\begin{aligned}\mathrm{E}[S_t] = {} & S_0 \exp\left\{\mu t - \frac{1}{2}\sigma^2 t^{2HK}\right\} \mathrm{E}[\exp\{\sigma B_t^{H,K}\}] = \\ & S_0 \exp\left\{\mu t - \frac{1}{2}\sigma^2 t^{2HK}\right\} \exp\{\sigma^2 E(B_t^{H,K})\} = \\ & S_0 \exp\left\{\mu t - \frac{1}{2}\sigma^2 t^{2HK}\right\} \exp\left\{\frac{1}{2}\sigma^2 t^{2HK}\right\} = S_0 \exp\{\mu t\},\end{aligned}$$

从而可得结果.

2 再装期权定价公式

考虑在到期日 T 之前只再装一次的情形,设 X 为 T 时刻的执行价格,再装期权收益结构如下:在再装日 $T_1(0 < T_1 < T)$ 的收益为 $V_{T_1} = (S_{T_1} - X)^+$,在到期日 T 的收益为

$$V_T = \begin{cases} \dfrac{X}{S_{T_1}}(S_T - S_{T_1})^+ = X\left(\dfrac{S_T}{S_{T_1}} - 1\right)^+, & S_{T_1} > X, \\ (S_T - X)^+, & S_{T_1} < X. \end{cases}$$

当 $T_1 \to T$ 时,再装期权即为欧式看涨期权.

定义 3[9] 再装期权保险精算价格定义为

$$\begin{aligned} V_0 = {} & \mathrm{E}\{[\exp\{-\beta T_1\}S_{T_1} - \exp\{-rT_1\}X] I_{\{\exp\{-\beta T_1\}S_{T1} > \exp\{-rT_1\}X\}}\} + \\ & \mathrm{E}\left\{X\left[\frac{\exp\{-\beta T\}S_T}{\exp\{-\beta T_1\}S_{T_1}} - \exp\{-rT_1\}\right] I_{\{\exp\{-\beta T_1\}S_{T1} > \exp\{-rT_1\}X,\ \frac{\exp\{-\beta T\}S_T}{\exp\{-\beta T_1\}S_{T1}} > \exp\{-rT_1\}\}}\right\} + \\ & \mathrm{E}\{[\exp\{-\beta T\}S_T - \exp\{-rT\}X] I_{\{\exp\{-\beta T_1\}S_{T1} < \exp\{-rT_1\}X,\ \exp\{-\beta T\}S_T > \exp\{-rT\}X\}}\}. \end{aligned} \tag{5}$$

其中风险资产按期望收益率折现,无风险资产按无风险利率折现.

定理 1 再装期权保险精算价格

$$\begin{aligned} V_0 = {} & \sum_{m=0}^{+\infty} \frac{\exp(-\lambda T_1)(\lambda T_1)^m}{m!} \{S_0 N(-d_1^{(m)} + \sigma_1^{(m)}) - X\exp\{-rT_1\}N(-d_1^{(m)})\} + \\ & X\sum_{m=0}^{+\infty}\sum_{n=0}^{+\infty} \frac{\exp(-\lambda T)(\lambda T_1)^m[\lambda(T - T_1)]^n}{m!\ n!}\{N(\rho_1^{(m,n)}\ \sigma_2^{(n)} - d_1^{(m)}, \sigma_2^{(n)} - d_2^{(n)}, \rho_1^{(m,n)}) - \\ & \exp\{-rT_1\}N(-d_1^{(m)}, -d_2^{(n)}, \rho_1^{(m,n)})\} + \\ & \sum_{m=0}^{+\infty}\sum_{n=0}^{+\infty} \frac{\exp(-\lambda T)(\lambda T_1)^m[\lambda(T - T_1)]^n}{m!\ n!}\{S_0 N(d_1^{(m)} - \rho_2^{(m,n)}\sigma^{(m,n)}, \sigma^{(m,n)} - c^{(m,n)}, -\rho_2^{(m,n)}) - \\ & X\exp\{-rT\}N(d_1^{(m)}, -c^{(m,n)}, -\rho_2^{(m,n)})\}. \end{aligned}$$

其中

$$d_1^{(m)} = \frac{\left\{\ln\left(\dfrac{X}{S_0}\right) - rT_1 + \dfrac{1}{2}\sigma_J^2\ T_1^{2HK} + \dfrac{m\sigma_J^2}{2}\right\}}{\sigma_1^{(m)}}, \quad \sigma_1^{(m)} = \sqrt{\sigma^2 T_1^{2HK} + m\sigma_J^2},$$

$$d_2^{(n)}=\frac{\left\{-rT+\frac{1}{2}\sigma^2(T^{2HK}-T_1^{2HK})+n\sigma_J^2\right\}}{\sigma_2^{(n)}},\quad \sigma_2^{(n)}=\sqrt{\sigma^2(T^{2HK}-T_1^{2HK})+n\sigma_J^2},$$

$$c^{(m,n)}=\frac{\left\{\ln\left(\frac{X}{S_0}\right)-rT_1+\frac{1}{2}\sigma^2T_1^{2HK}+\frac{(m+n)\sigma_J^2}{2}\right\}}{\sigma^{(m,n)}},\quad \sigma^{(m,n)}=\sqrt{\sigma^2T^{2HK}+(m+n)\sigma_J^2},$$

$$\rho_1^{(m,n)}=\frac{\{\sigma^2(T^{2HK}-T_1^{2HK}-(T-T_1)^{2HK})\}}{\{2\sigma_1^{(m)}\sigma_2^{(n)}\}},$$

$$\rho_2^{(m,n)}=\frac{\{\sigma^2(T^{2HK}+T_1^{2HK}-(T-T_1)^{2HK}+2m\sigma_J^2)\}}{\{2\sigma_1^{(m)}\sigma^{(m,n)}\}},$$

$$\varphi(u)=\frac{1}{\sqrt{2\pi}}\exp\left\{\frac{-u^2}{2}\right\},\varphi(u,v,\rho)=\frac{1}{2\pi\sqrt{1-\rho^2}}\exp\left\{\frac{-\{u^2-2\rho uv+v^2\}}{\{2(1-\rho^2)\}}\right\},$$

$$N(x)=\int_{-\infty}^{x}\varphi(u)\mathrm{d}u,\quad N(x,y,\rho)=\int_{-\infty}^{x}\int_{-\infty}^{y}\varphi(u,v,\rho)\mathrm{d}u\mathrm{d}v.$$

证明　记

$$V_1=\mathrm{E}\{[\exp\{-\beta T_1\}S_{T_1}-\exp\{-rT_1\}X]I_{\{\exp\{-\beta T_1\}S_{T1}>\exp\{-rT_1\}X\}}\},$$

$$V_2=\mathrm{E}\left\{X\left[\frac{\exp\{-\beta T\}S_T}{\exp\{-\beta T_1\}S_{T_1}}-\exp\{-rT_1\}\right]I_{\{\exp\{-\beta T_1\}S_{T1}>\exp\{-rT_1\}X,\ \frac{\exp\{-\beta T\}S_T}{\exp\{-\beta T_1\}S_{T1}}>\exp\{-rT_1\}\}}\right\},$$

$$V_3=\mathrm{E}\{[\exp\{-\beta T\}S_T-\exp\{-rT\}X]I_{\{\exp\{-\beta T_1\}S_{T1}<\exp\{-rT_1\}X,\ \exp\{-\beta T\}S_T>\exp\{-rT\}X\}}\},$$

假设在区间 $[0,T_1]$ 内发生跳跃 $J_1(i),i=1,2,\cdots,m$,在 $[T_1,T]$ 内发生跳跃 $J_2(i)$,$i=1,2,\cdots,n$.

(1) 计算 V_1,由于

$$\xi=\frac{\left\{\sigma B_{T_1}^{H,K}+\sum_{i=1}^{m}J_1(i)+\frac{m\sigma_J^2}{2}\right\}}{\sigma_1^{(m)}}\sim N(0,1),$$

$$\{S_{T_1}\exp\{-\beta T_1\}>X\exp\{-rT_1\}\}=\{\xi>d^{(m)}\},$$

$$S_{T_1}=S_0\exp\left\{\mu T_1-\frac{1}{2}\sigma^2T_1^{2HK}-\frac{m\sigma_J^2}{2}+\sigma_1^{(m)}\xi\right\},$$

则

$$\begin{aligned}V_1=&\mathrm{E}\{\{S_{T_1}\exp\{-\mu T_1\}-X\exp\{-rT_1\}\}I_{\{\xi>d_1^{(m)}\}}\}=\\&\mathrm{E}\{\mathrm{E}\{\{S_{T_1}\exp\{-\mu T_1\}-X\exp\{-rT_1\}\}I_{\{\xi>d_1^{(m)}\}}\mid Q_{T_1}\}\}=\\&\sum_{m=0}^{+\infty}\frac{\exp(-\lambda T)(\lambda T_1)^m}{m!}\{E\{S_{T_1}\exp\{-\mu T_1\}I_{\{\xi>d_1^{(m)}\}}\mid Q_{T_1}=m\}-\\&X\exp\{-rT_1\}E\{I_{\{\xi>d_1^{(m)}\}}\mid Q_{T_1}=m\}\}=\\&\sum_{m=0}^{+\infty}\frac{\exp^{-\lambda T}(\lambda T)^m}{m!}(V_{11}-V_{12}).\end{aligned}$$

其中

$$\begin{aligned}V_{11}=&\mathrm{E}\{S_{T_1}\exp\{-\mu T_1\}I_{\{\xi>d_1^{(m)}\}}\mid Q_{T_1}=m\}=\\&\mathrm{E}\{S_0\exp\left(-\frac{1}{2}\sigma^2T_1^{2HK}-\frac{m\sigma_J^2}{2+\sigma_1^{(m)}\xi}\right)I_{\{\xi>d_1^{(m)}\}}\}=\\&\mathrm{E}\{S_0\int_{d_1^{(m)}}^{+\infty}\exp\left\{-\frac{(\sigma_1^{(m)})^2}{2}+\sigma_1^{(m)}u\right\}\frac{1}{\sqrt{2\pi}}\exp\left\{-\frac{u^2}{2}\right\}\mathrm{d}u\}=\\&S_0N(-d_1^{(m)}+\sigma_1^{(m)}),\end{aligned}$$

$$\begin{aligned}V_{12}=&X\exp\{-rT_1\}\mathrm{E}\{I_{\{\xi>d_1^{(m)}\}}\mid Q_{T_1}=m\}=\\&X\exp\{-rT_1\}P\{\xi>d_1^{(m)}\}=\\&X\exp\{-rT_1\}N(-d_1^{(m)}).\end{aligned}$$

(2) 计算 V_2,由于

$$\eta=\frac{\left\{\sigma(B_T^{H,K}-B_{T_1}^{H,K})+\sum_{i=1}^{n}J_2(i)+\frac{n\sigma_J^2}{2}\right\}}{\sigma_2^{(n)}}\sim N(0,1),$$

$$\left\{\frac{S_T}{S_{T_1}}\exp\{-\mu(T-T_1)>\exp\{-rT_1\}\right\}=\{\eta>d_2^{(n)}\},$$

$$\frac{S_T}{S_{T_1}}=\exp\left\{\mu(T-T_1)-\frac{1}{2}\sigma^2(T^{2HK}-T_1^{2HK})-\frac{n\sigma_J^2}{2}+\sigma_2^{(n)}\ \eta\right\},$$

$$\rho_{\xi,\eta}=\mathrm{E}(\xi\eta)=\rho_1^{(m,n)},$$

则

$$V_2=X\mathrm{E}\left\{\left\{\frac{S_T}{S_{T_1}}\exp\{-\mu(T-T_1)\}-\exp\{-rT_1\}\right\}\right.$$
$$\left.I_{\exp\{-\mu T_1\}S_{T1}>\exp\{-rT_1\}X,\ \frac{\exp\{-\mu T\}S_T}{\exp\{-\mu T_1\}S_{T1}}>\exp\{-rT_1\}}\right\}=$$
$$X\mathrm{E}\left\{\mathrm{E}\left\{\left\{\frac{S_T}{S_{T_1}}\exp\{-\mu(T-T_1)\}-\exp\{-rT_1\}\right\}\right.\right.$$
$$\left.\left.I_{\exp\{-\mu T_1\}S_{T1}>\exp\{-rT_1\}X,\ \frac{\exp\{-\mu T\}S_T}{\exp\{-\mu T_1\}S_{T1}}>\exp\{-rT_1\}}\right\}\mid Q_{T_1},Q_{T-T_1}\right\}=$$
$$X\sum_{m=0}^{+\infty}\sum_{n=0}^{+\infty}\frac{\exp(-\lambda T)(\lambda T_1)^m[\lambda(T-T_1)]^n}{m!\ n!}\cdot$$
$$\left\{\mathrm{E}\{\frac{S_T}{S_{T_1}}\exp\{-\mu(T-T_1)\}I_{\{\xi>d_1^{(m)},\eta>d_2^{(n)}\}}\mid Q_{T_1}=m,Q_{T-T_1}=n\right\}-$$
$$\exp\{-rT_1\}\mathrm{E}\{I_{\{\xi>d_1^{(m)},\eta>d_2^{(n)}\}}\mid Q_{T_1}=m,Q_{T-T_1}=n\}\}=$$
$$X\sum_{m=0}^{+\infty}\sum_{n=0}^{+\infty}\frac{\exp(-\lambda T)(\lambda T_1)^m[\lambda(T-T_1)]^n}{m!\ n!}\{V_{21}-V_{22}\}.$$

其中

$$V_{21}=\mathrm{E}\{\frac{S_T}{S_{T_1}}\exp\{-\mu(T-T_1)\}I_{\{\xi>d_1^{(m)},\eta>d_2^{(n)}\}}\mid Q_{T_1}=m,Q_{T-T_1}=n\}=$$
$$\exp\left\{-\frac{1}{2}(\sigma_2^{(n)})^2\right\}\int_{d_1^{(m)}}^{+\infty}\int_{d_2^{(n)}}^{+\infty}\exp\{\sigma_2^{(n)}\ v\}\phi(u,v,\rho_1^{(m,n)})\mathrm{d}u\,\mathrm{d}v=$$
$$\int_{d_1^{(m)}}^{+\infty}\int_{d_2^{(n)}}^{+\infty}\phi(u-\rho_1^{(m,n)}\ \sigma_2^{(n)},v-\sigma_2^{(n)},\rho_1^{(m,n)})\mathrm{d}u\,\mathrm{d}v=$$
$$\int_{d_1^{(m)}-\rho_1^{(m,n)}\sigma_2^{(n)}}^{\infty}\int_{d_2^{(n)}-\sigma_2^{(n)}}^{\infty}\phi(x,y,\rho_1^{(m,n)})\mathrm{d}x\,\mathrm{d}y=$$
$$N(\rho_1^{(m,n)}\ \sigma_2^{(n)}-d_1^{(m)},\sigma_2^{(n)}-d_2^{(n)},\rho_1^{(m,n)}),$$
$$V_{22}=\exp\{-rT_1\}\mathrm{E}\{I_{\{\xi>d_1^{(m)},\eta>d_2^{(n)}\}}\mid Q_{T_1}=m,Q_{T-T_1}=n\}=$$
$$\exp\{-rT_1\}N(-d_1^{(m)},-d_2^{(n)},\rho_1^{(m,n)}).$$

（3）计算 V_3，由于

$$\gamma=\frac{\left\{\sigma B_T^{H,K}+\sum_{i=1}^{m}J_1(i)+\sum_{i=1}^{n}J_2(i)+\frac{(m+n)\sigma_J^2}{2}\right\}}{\sigma^{(m,n)}}\sim N(0,1),$$

$$\{S_{T_1}\exp(-\mu T_1)<X\exp\{-rT_1\}\}=\{\xi<d_1^{(m)}\},$$

$$\{S_T\exp\{-\mu T\}>X\exp\{-rT\}\}=\{\gamma>c^{(m,n)}\},$$

$$S_T=S_0\exp\left\{\mu T-\frac{1}{2}\sigma^2T^{2HK}-\frac{(m+n)\sigma_J^2}{2+\sigma^{(m,n)}\gamma}\right\},$$

$$\rho_{\xi,\gamma}=\mathrm{E}(\xi\gamma)=\rho_2^{(m,n)},$$

则

$$V_3=\mathrm{E}\{\{S_T\exp\{-\mu T\}-X\exp\{-rT\}\}I_{\{\exp\{-\mu T_1\}S_{T1}<\exp\{-rT_1\}X,\ \exp\{-\mu T\}S_T>\exp\{-rT\}X\}}\}=$$

$$\sum_{m=0}^{+\infty}\sum_{n=0}^{+\infty}\frac{\exp(-\lambda T)(\lambda T_1)^m[\lambda(T-T_1)]^n}{m!\ n!}\{E\{S_T\exp\{-\mu T\}I_{\{\xi<d_1^{(m)},\gamma>c^{(m,n)}\}}\mid Q_{T_1}=m,Q_{T-T_1}=n\}-$$

$$X\exp\{-rT\}\mathrm{E}\{I_{\{\xi<d_1^{(m)},\gamma>c^{(m,n)}\}}\mid Q_{T_1}=m,Q_{T-T_1}=n\}=$$

$$\sum_{m=0}^{\infty}\sum_{n=0}^{\infty}\frac{\exp(-\lambda T)(\lambda T_1)^m[\lambda(T-T_1)]^n}{m!\ n!}\{V_{31}-V_{32}\}.$$

其中

$$\begin{aligned}V_{31}=&\mathrm{E}\{S_T\exp\{-uT\}I_{\xi<d_1^{(m)},\gamma>c^{(m,n)}}\mid Q_{T_1}=m,Q_{T-T_1}=n\}=\\&\mathrm{E}\{S_0\exp\left\{-\frac{1}{2}(\sigma^{(m,n)})^2+\sigma^{(m,n)}\gamma\right\}I_{\{\xi<d_1^{(m)},\gamma>c^{(m,n)}\}}\}=\\&S_0\exp\left\{-\frac{1}{2}(\sigma^{(m,n)})^2\right\}\int_{-\infty}^{d_1^{(m)}}\int_{c(m,n)}^{+\infty}\exp\{\sigma^{(m,n)}v\}\phi(u,v,\rho_2^{(m,n)})\mathrm{d}u\mathrm{d}v=\\&S_0\int_{-\infty}^{d1(m)-\rho_2^{(m,n)}\sigma(m,n)}\int_{c(m,n)-\sigma(m,n)}^{+\infty}\phi(x,y,\rho_2^{(m,n)})\mathrm{d}x\mathrm{d}y=\\&S_0N(d_1^{(m)}-\rho_2^{(m,n)}\sigma^{(m,n)},\sigma^{(m,n)}-c^{(m,n)},-\rho_2^{(m,n)}),\\V_{32}=&X\exp\{-rT\}\mathrm{E}\{I_{\{\xi<d_1^{(m)},\gamma>c^{(m,n)}\}}\mid Q_{T_1}=m,Q_{T-T_1}=n\}=\\&X\exp\{-rT\}P\{\xi<d_1^{(m)},\gamma>c^{(m,n)}\}=\\&X\exp\{-rT\}N(d_1^{(m)},-c^{(m,n)},-\rho_2^{(m,n)}).\end{aligned}$$

3　结　论

当金融市场中出现标的资产不平稳发展时，分数布朗运动无法对金融市场准确描述，而双分数布朗运动却可以描述不平稳发展的金融市场.

（1）当 $\lambda=0$ 时，可得双分数布朗运动环境下再装期权定价公式.当 $K=1$ 时，可得分数布朗运动环境下再装期权定价公式.

（2）当 $T_1\to T$ 时，可得双分数扩散-跳散过程下欧式看涨期权价格

$$c=\sum_{n=0}^{+\infty}\frac{\exp(-\lambda T)(\lambda T)}{n!}\{S_0N(d_1^{(n)})-X\exp\{-rT\}N(d_2^{(n)})\}.$$

其中 $N(x)$ 为标准正态分布函数，且

$$d_1^{(n)}=\frac{\ln\left(\frac{S_0}{X}\right)+rT+\frac{1}{2}(\sigma^2T^{2HK}+n\sigma_J^2)}{\sqrt{\sigma^2T^{2HK}+n\sigma_J^2}},\quad d_2^{(n)}=d_1^{(n)}-\sqrt{\sigma^2T^{2HK}+n\sigma_J^2}.$$

特别地，当 $\lambda=0$ 时，可得双分数布朗运动环境下欧式看涨期权定价公式

$$c=S_0N(d_1)-X\exp\{-rT\}N(d_2).$$

其中

$$d_1=\frac{\ln(S_0/X)+rT+\frac{1}{2}\sigma^2T^{2HK}}{\sigma T^{HK}},d_2=d_1-\sigma T^{HK}.$$

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